Seat No. : _____

N16-102

November-2014

B.Sc., Sem.-V

Statistics, Paper-STA-304

(Exact Sampling Distributions and their applications)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory and carry equal marks.
(2) Statistical tables and graph papers will be provided on request.
(3) Use of Scientific calculator is allowed.

1. (a) For Chi-square distribution with n degrees of freedom, derive the moment generating function.

OR

State and prove the additive property of Chi-square distribution.

(b) If X and Y be two independent random variables following chi square distribution with m and n degrees of freedom respectively then derive the distribution of X / (X + Y).

OR

If X is a Chi Square variate with 4 degrees of freedom, then for a > 0, show that

$$P(X \geq a) = \left(e^{\frac{-a}{2}}\right)\left(1+\frac{a}{2}\right)$$

2. (a) Obtain the first three raw moments of t-distribution.

OR

Derive the probability distribution of t-according to Fisher.

(b) Derive the probability distribution of sample correlation coefficient.

OR

Establish a test procedure of testing the significance of single sample mean of a random sample of size n.

3. (a) Define Snedocor's F-distribution. State its applications.

OR

In usual notations, prove that the mode of the Snedocor's F-distribution is always less than unity.

(b) Establish the relation between F-distribution and Chi-square distributions.

OR

Explain a test procedure to test the significance of observed sample correlation coefficient r, when a value of population correlation coefficient is non zero.

4. (a) Define compound distribution. State its importance.

OR

Derive negative binomial distribution as a compound distribution of Poisson distribution and gamma distribution.

(b) What is the compound binomial distribution ? Obtain its probability function and identify it.

OR

If a random variable X follows Poisson distribution with parameter m and the parameter m, if treated as a random variable following the probability density function

$f(m) = \frac{a^{\lambda}}{\sqrt{\lambda}} e^{-am} m^{(\lambda - 1)}$, $m > 0$, $a > 0$, $\lambda > 0$, then obtain the probability function of X.

5. Answer the following, in brief :

(a) If X and Y be two independent r.v.s. following chi square distribution with m and n degrees of freedom respectively, then write the probability distribution of X/Y and state it's mean and variance.

(b) State test statistics according to Student and Fisher.

(c) State a test statistic which is used to test the null hypothesis $H_0 : \rho = 0$

(Where ρ is a population correlation coefficient). If sample correlation coefficient of a paired sample of size 15 is 0.48, do you agree with its significance at 5% level ?

(d) If a random variable X has Chi square distribution with 2, then state the value of β_2.

(e) If a random variable X follows a Chi square distribution with 4 degrees of freedom, find the skewness according to Karl Pearson.

(f) If a random variable X has F-distribution with (m, n) degrees of freedom, and if a transformation $Y = \frac{1}{2} \log_e X$, then state the probability distribution of Y.

(g) If a random variable X has F-distribution with (m, n) degrees of freedom, then state the distribution of 1/X and give one use of this relationship.

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